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Ideal MHD Properties for Proposed Noncircular Tokamaks*

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We obtain Double Dee, TFCX-C, Big Dee, and JET equilibria which are optimized with respect to both shape and current profile for stability to ideal MHD modes. With a wall reasonably far from the plasma surface we find that the external kink constrains q_l to be above two, where q_l is the plasma surface value of the safety factor, and the ballooning mode limits the value of β. Then a relevant stable β value for the Double Dee reactor design is over 7%. Such a Double Dee equilibrium is not in a separated second stability region and thus does not have a problem with accessibility. A relevant stable β value for the TFCX-C reactor design is over 6%. Equivalent relevant stable β values for the Big Dee (17%) and JET (7%) are included for calibration purposes. We compare these relevant stable β values with the β's determined by two recent scaling laws.

1. INTRODUCTION

Using D-D reaction, tokamak reactor designs become economically attractive when β, the ratio of volume averaged plasma to magnetic pressure exceeds 5%. Ideal MHD instabilities are of great concern because they have the potential to limit β below this value and so extensive studies have been done to determine ideal MHD β limits [1–5]. As the maximum stable value of β, to be denoted β_c, increases with inverse aspect ratio ε, elongation κ, and triangularity δ, the Double Dee reactor design (see Fig. 1) is particularly suited to obtain high values of β_c.

Assuming no wall stabilization, the $n = 1$ external kink is the most unstable mode [6]. However, this $n = 1$ external kink limit has been marginally exceeded in Doublet III [7] and clearly exceded in PDX [8]. Thus the usual theoretical beta limits imposed by ideal MHD stability have been violated. An explanation for this discrepancy is that the vacuum wall, though resistive, can appear superconducting for rotating modes. Wall stabilization is realistic if the real frequency of the kink is much greater than the inverse resistive diffusion time of the vacuum vessel [9]. Through comparison of ideal MHD β limits with recent Doublet III experimental data it has been determined [7] that for $q_l > 2$ (q_l is the safety factor at the plasma

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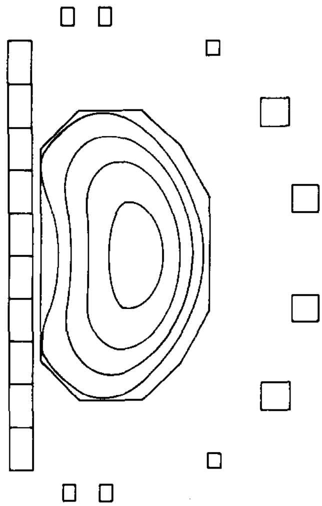

FIG. 1. Double Dee design and equilibrium with $\beta = 7.4\%$ and $q_l = 2.01$.

surface), β is limited by the ballooning mode if the wall is reasonably far from the plasma surface ($d/a \leqslant 1.5$, where d and a are the wall and plasma radii, respectively). On the other hand, for $q_l \leqslant 2$, the $n = 1$ external kink is unstable even with a wall fairly close by. That is, the external kink limits $q_l > 2$ and the ballooning mode limits the value of β.

Thus, to obtain a relevant β limit for the Double Dee reactor design and the TFCX-C reactor design it is appropriate to obtain equilibria stable to ballooning modes with $q_l > 2$. Then, stable Double Dee equilibria with β's over 7% are obtained with dee-shaped plasmas with small inside indentation (see Fig. 1 for such

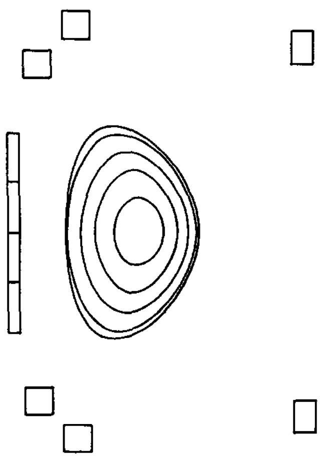

FIG. 2. TFCX-C design and equilibrium with $\beta = 6\%$ and $q_l = 2.00$.

an equilibrium). These equilibria are not in a separated second stability region and hence do not have a problem with accessibility. The achievement of such β values makes possible advanced fuel reactions (D-D and D-He). TFCX-C equilibria stable to ballooning modes with $q_l > 2.0$ are obained with β's of 6.0% (see Fig. 2 for such an equilibrium). Such equilibria are examined in detail and then compared with similar equilibria obtained for the Big Dee and JET. The β's obtained for the four devices are then compared with the β's predicted by two scaling laws, the scaling law of Sykes, Turner, and Patel [10, 11] and the scaling law of Bernard, Helton, Moore, and Todd [7].

In Section 2, details of the numerical analysis are discussed. In Section 3, the results of the stability analysis are presented. In Section 4, these β limits are compared with the β's predicted by the two scaling laws. In Section 5, $n = 0$, 1, 2, and 3 kink modes are discussed for the four devices. In Section 6, the main results in the paper are summarized and discussed.

2. Numerical Techniques

The maximum stable value of β that can be achieved for a given configuration (β_c) can be increased through optimization of cross-sectional shape and current profile. Numerical techniques have been developed for determining ideal MHD stability and for optimizing cross-sectional shape and current profile. These numerical techniques have been combined with realistic current profiles and used to obtain symmetric free-boundary equilibria stable to ideal modes for poloidal field-shaping coil sets that are specific to the Double Dee, TFCX-C, Big Dee, and JET devices.

The equilibrium calculations were done using GAEQ [12], a realistic ideal MHD equilibrium code which permits careful modeling of physical details such as limiter position, coil cross-secton, magnetic diagnostics, etc. The MHD equilibria are solutions to the Grad–Shafranov equation

$$R^2\nabla\cdot\left(\frac{\nabla\psi}{R^2}\right) = -\mu_0 R j_\phi = -\mu_0 R^2 p' - ff', \tag{1}$$

with current profiles of the form $j_\phi = C[b_p h_p(\tilde{\psi})\, R/R_0 + (1 - b_p)\, h_f(\tilde{\psi})\, R_0/R]$, where $\tilde{\psi} = (\psi - \psi_0)/(\psi_l - \psi_0)$. $2\pi\psi_0$ and $2\pi\psi_l$ refer to the flux values at the magnetic axis and limiter, respectively, and q_0 and q_l refer to the safety factor at the magnetic axis and limiter, respectively. The constant C is used to satisfy the integral constraint that the total plasma current be a specified constant. R_0 is the radius of the center of the midplane limiter and b_p is used to vary the poloidal beta β_p. The function $h_p(\tilde{\psi}) = [\exp(1 - \tilde{\psi}^{\alpha_p}) - 1.]/(\exp(1.) - 1.)$ will be referred to as the exponential p' profile. Then α_p is a measure of the width of the p' profile, with increasing α_p corresponding to broader profiles. Similarly, for the exponential profile $h_f(\tilde{\psi}) =$

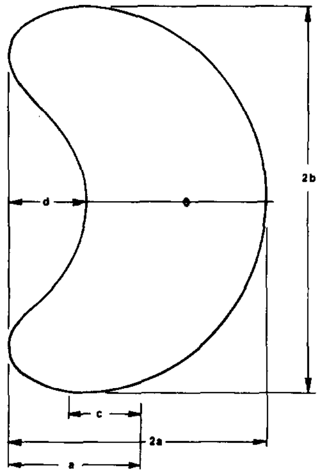

FIG. 3. Definitions of elongation, triangularity, and indentation.

$[\exp(1-\bar{\psi}^{\alpha_f})-1.]/[\exp(1.)-1.]$, α_f is used to describe the variation of the toroidal field.

Beta is defined to be $2\mu_0 \int p\, dV/B_T^2$, where B_T is the vacuum toroidal field measured at R_0. The definition used for beta poloidal: $\beta_{ps} = (1/2\pi) \int p\, dV / \int (V/V')\, I'\, d\psi$ is the expression introduced by Shafranov [12], where V is the volume and I the toroidal current. With reference to Fig. 3, elongation denoted κ is defined to be b/a, triangularity denoted δ is defined to be c/a and indentation i is defined to be $d/2a$.

The following numerical techniques were used to determine stability to ideal MHD modes. Stability to high toroidal mode number was analyzed using the code MBC [14] which evaluates the ballooning criterion in the limit of infinite toroidal mode number. A critical value of β for marginal stability to ballooning modes is calculated by scaling the toroidal field and hence q_0 to the values at which the equilibria are marginally stable and then calculating β at that toroidal field. Stability to low toroidal mode number perturbations was analyzed using the global stability code GATO [15]. GATO computes the eigenfrequencies and eigenmodes using finite hybrid elements to minimize the symmetric form of δW.

An equilibrium is determined by a set of parameters: current profile parameters, b_p α_p, and α_f and shape parameters which are the flux values on the field-shaping coils. Starting from a stable initial equilibrium, an equilibrium optimized for stable beta is obtained by varying in turn shape and current profile parameters and retaining favourable variants subject to certain engineering and other constraints. Favorable variants are those for which the present marginal stable β, as determined by the scaling defined earlier, is greater than the previous marginal stable β. Clearly this iterative process can determine only a local maximum in β. The initial equilibrium for each optimization has shape parameters close to the stated geometry of the device. In the course of the optimization the shape parameters are somewhat constrained by the shape parameters of the limiter.

Determination of the exact value of β_c for external modes using mesh extrapolation requires large amounts of computer time and is therefore impractical for these optimization studies. Instead, an equilibrium is said to be stable if the growth rate ω is less than a stability cutoff $(10^{-2}\omega_A)$, where ω_A is the poloidal Alfvén frequency. After an optimization is completed, some resulting equilibria are examined in detail to ensure stability to external modes by computing the growth rate with extrapolation to zero mesh size.

3. Stability Results

Equilibrium and stability studies have been done for the Double Dee, TFCX-C, and JET. Earlier results [5] from equivalent calculations done for the Big Dee have been included for comparison purposes.

3.1. *Double Dee Reactor*

This device has twenty coils, $R_0 = 3.31m$, $\alpha = 1.39m$, $\varepsilon = 0.42$, and the toroidal current is 10 MA. The coil and limiter design can be seen in Fig. 1. Because of the geometry of the Double Dee, small aspect ratio, large height to width ratio, and significant triangularity, it is expected that there will exist symmetric dee equilibria in the device stable to ideal modes at high values of β. In fact, using very simple current profiles, there exist high β equilibria with a plasma surface shape similar to the shape of the limiter. The equilibria described in this paper were obtained using the automated shape and current profile optimization code.

Equilibrium stable to ballooning modes only with no restriction on q_l are the most optimistic cases. If such cases are to make any sense some mechanism such as a cold plasma mantle between the plasma surface and the limiter such that the plasma would be in effect surrounded by a perfect conductor must be hypothesized. Alternatively, the limiter itself could be considered to have an important stabilizing effect [16].

As discussed in detail above, recent calculations indicate that the equilibria stable to ballooning modes with $q_l > 2$ are the equilibria which will be close to the experimental beta limit and hence warrant further study. It can be shown that these equilibria are not in a separated second stability region [17]. In other recent work [18] it was shown that for low values of q_l (< 3) there is no second stability region as a function of indentation.

With $q_l > 2$ the highest stable β equilibrium obtained for the Double Dee had $\beta = 7.4\%$, $q_0 = 1.32$, $q_l = 2.01$, $B_T = 2.32$ T at 10 MA. With $q_l < 2$ the highest stable β equilibrium obtained for the Double Dee had $\beta = 10.4\%$, $q_0 = 1.10$, $q_l = 1.41$, $B_T = 1.67$ T at 10 MA.

The two cases described above have increasing β stable to ballooning modes. The elongation and indentation increase as β increases and q_l decreases. The evolution

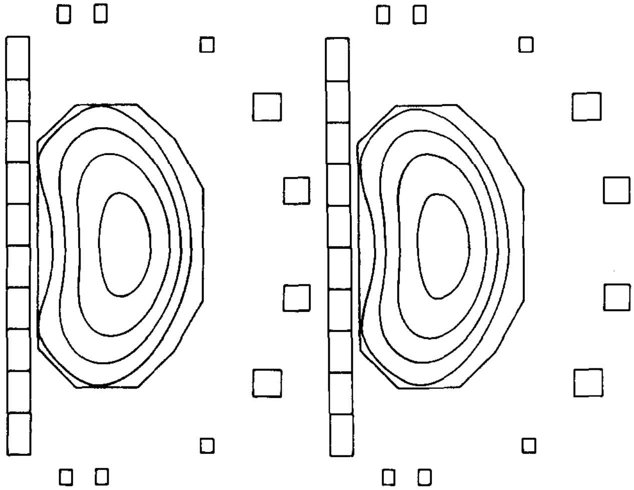

FIG. 4. Evolution of Double Dee equilibrium shape as β_c increases from 7.4 to 10.4%.

of the shape is shown in Fig. 4. The evolution of the q profile is shown in Fig. 5. The optimization was stopped because of computer time limitations but it appears that this process would have continued generating equilibria with increasing β stable to ballooning modes, increasing elongation and identation, decreasing q_l and with further evolution of the shape and q profile as indicated in Fig. 4 and 5. This region of stability has not been investigated before.

3.2. TFCX-C *Reactor*

This design has ten coils. The limiter points are given by:

$$r = R_0 + a\cos(\theta + \delta\sin\theta),$$

$$z = a\kappa\sin\theta,$$

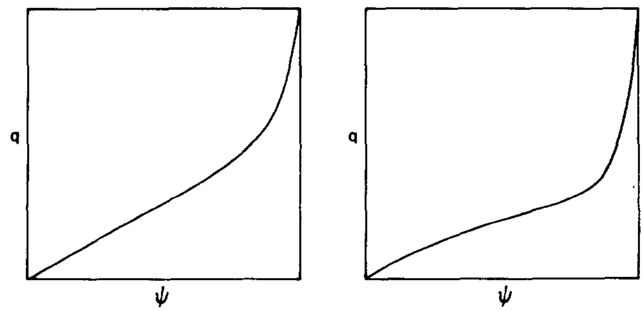

FIG. 5. Evolution of Double Dee q profile as β_c increases from 7.4 to 10.4%.

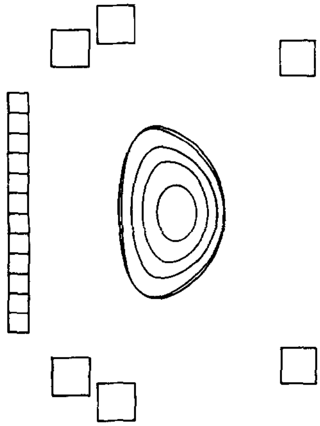

FIG. 6. TFCX-H design and equilibrium with $\beta = 4.0\%$ and $q_l = 2.00$.

where $R_0 = 3.00$ m, $a = 1.20$ m, $\delta = 0.3$, and $\kappa = 1.6$. $\varepsilon = 0.40$. The formula above was supplied by Dennis Strickler. The coil and limiter design can be seen in Fig. 2. With $q_l > 2$, the highest stable β equilibrium obtained for TFCX-C had $\beta = 6.0\%$, $q_0 = 1.16$, $q_l = 2.00$, $B_T = 4.30$ T at 11.0 MA. With $q_l < 2$, an equilibrium was obtained with $\beta = 7.2\%$, $q_0 = 1.17$, $q_l = 1.89$, $B_T = 3.97$ T at 11.0 MA.

TFCX-H was also studied. This design has eighteen coils. The limiter points are given by the formula above where $R_0 = 3.75$ m, $a = 1.07$ *m*, $\delta = 0.3$, and $\kappa = 1.6$. $\varepsilon = 0.29$. The positions of the poloidal field coils were taken from Fig. 7 in Ref. [19].

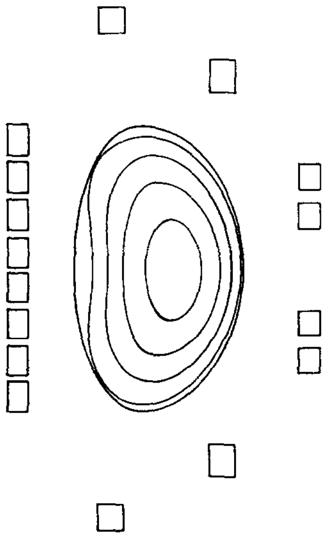

FIG. 7. JET device and equilibrium with $\beta = 7.4\%$ and $q_l = 2.00$.

The limiter design can also be seen in Fig. 6. With $q_l > 2$, the highest stable β equilibrium obtained for TFCX-H had $\beta = 4.0\%$, $q_0 = 1.05$, $q_l = 2.00$, $B_T = 4.44$ T at 7.7 MA. With $q_l < 2$, an equilibrium was obtained with $\beta = 5.0\%$, $q_0 = 1.05$, $q_l = 1.60$, $B_T = 3.91$ T at 7.7 MA. These β values could not be obtained with the optimization code but required extensive iterative manipulation of the TFCX-H shape and current profile. This difficulty could indicate the existence of important design considerations. To obtain optimum β values the shaping coils should probably be reasonable close to the plasma.

3.3. JET

This devise has sixteen coils. The limiter points are given by:

$$z^2 = \frac{43.8 - 0.8(r^2 - 10.32)^2}{1 + r^2}.$$

The positions of the poloidal field coils were taken from p. 186 of Ref. [20]. The coil and limiter design can be seen in Fig. 7. With $q_l > 2$, the highest stable β equilibrium obtained for JET had $\beta = 7.4\%$, $q_0 = 1.22$, $q_l = 2.00$, $B_T = 1.53$ T at 4.8 MA. With $q_l < 2$, the highest stable β equilibrium obtained for JET had $\beta = 8.3\%$, $q_0 = 1.06$, $q_l = 1.90$, $B_T = 1.44$ T at 4.8 MA.

3.4. *The Big Dee*

This device has eighteen coils. $R_0 = 1.70$ m, $a = 0.72$ m, $\varepsilon = 0.42$, and δ and κ can vary. The coil and limiter design can be seen in Fig. 8. With $q_l > 2$, the highest

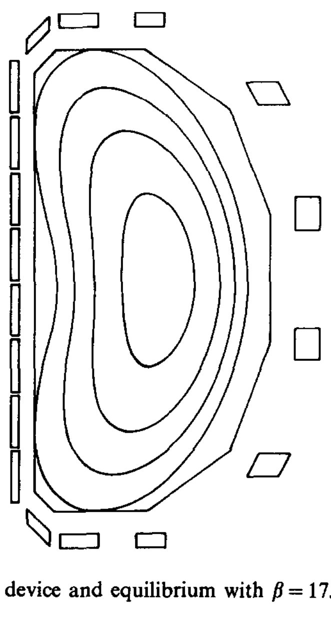

FIG. 8. Big Dee device and equilibrium with $\beta = 17.2\%$ and $q_l = 2.00$.

TABLE 1

Comparison of β_c with $\hat{\beta}^a$

	I_p (MA)	a (m)	B_T (T)	$\hat{\beta}$ (%)	β (%)
Double Dee	10.0	1.39	2.32	13.6	7.4
TFCX-C	11.0	1.20	4.30	9.4	6.0
JET	4.8	1.25	1.53	11.0	7.4
Big Dee	5.0	0.72	1.46	20.9	17.2

[a] Sykes, Turner, and Patel [10, 11].

stable β equilibrium obtained for the Big Dee had $\beta = 17.2\%$, $q_0 = 1.11$, $q_l = 2.00$, and $B_T = 1.46$ T at 5 MA. With $q_l < 2$, the highest stable β equilibrium obtained for the Big Dee had $\beta = 21.8\%$, $q_0 = 1.08$, $q_l = 1.76$, and $B_T = 1.25$ T at 5 MA. These results are taken from Ref. [5].

4. Scaling Laws

These β's can be compared with recent scaling laws. Taking for each of the four devices, the equilibrium with highest β with $q_l > 2$ and using the scaling law due to Sykes, Turner, and Patel [10, 11],

$$\hat{\beta} = \frac{3.5\mu_0 I_p}{aB_T},$$

we obtain the results in Table I.

Taking for each of the four devices the equilibrium with highest β with $q_l > 2$, and using the scaling law due to Bernard, Helton, Moore, and Todd [7],

$$\hat{\beta} = 27.0\varepsilon^{1.3}\kappa^{1.2}\frac{1.0 + 1.5\delta}{q_l^{1.1}},$$

we obtain the results in Table II.

Thus, the results obtained here are in general agreement with other experience.

TABLE II

Comparison of β_c with $\hat{\beta}^a$

	ε	κ	δ	q_l	$\hat{\beta}$ (%)	β (%)
Double Dee	0.40	1.95	0.24	2.01	11.2	7.4
TFCX-C	0.30	1.50	0.25	2.00	5.9	6.0
JET	0.42	1.73	0.20	2.00	9.9	7.4
Big Dee	0.40	2.15	0.45	2.00	16.1	17.2

[a] Bernard *et al.* [1].

TABLE III

Stability to Kink Modes for Selected Equilibria

	n			
	0	1	2	3
Double Dee	*s*	*u*	*u*	*u*
TFCX-C	*u*	*s*	*s*	*s*
JET	*s*	*u*	*u*	*u*
Big Dee	*s*	*s*	*s*	*s*

5. Kink Studies

Kink stability studies have been done for the Double Dee, TFCX-C, the Big Dee, and JET. For each of these four machines calculations and convergence studies were done using GATO to determine if an assumed superconducting wall through the center of the poloidal field-shaping coils of the device would suffice to stabilize the $n=0$, 1, 2, and 3 modes. For each of these devices, the equilibrium chosen for analysis was the equilibrium with highest β stable to ballooning modes with $q_l>2$. Table III summarizes the results of these studies.

These calculations could be continued with an assumed superconducting wall at various positions closer to the plasma.

6. Discussion

We have obtained Double Dee and TFCX-C equilibria optimized with respect to both shape and current profile for stability to ideal MHD modes. Our recent calculations indicate that a Double Dee equilibrium stable to ballooning modes at 7.4% with $q_l>2$ is the equilibrium which will be close to the experimental β limit. This equilibrium is not in a separated second stability region and hence does not have problems with accessibility. The achievement of such β values makes fusion reactors more credible. Ignoring kink modes, we have found a new high β bean stability region for Double Dee (see Figs. 4 and 5). This discovery suggests that it is possible to design bean reactors which require no internal pusher coil. A relevant β value for TFCX-C is 6% with $q_l>2$. A relevant stable β value for JET is 7.4% with $q_l>2$. A relevant stable β value for the Big Dee is 17% with $q_l>2$.

These β values are in agreement with both of the scaling laws we used, the scaling law due to Sykes, Turner, and Patel and the scaling law due to Bernard, Helton, Moore, and Todd. The agreement is somewhat better with the latter.

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